A MODEL OF DUAL ECONOMY

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I *Introduction*

It is a common feature in most under-developed countries of the present day that, while some industries being conducted under relatively highly capitalistic methods of production, there still remain, and sometimes to a considerable extent, many backward industries around the economy. Such a 'duality' in economic activity might be conceived of as being only a temporary phenomenon in the normal course of development in the capitalist economy, and indeed modern economists used to suppose that the whole economy is under the direct regime of capitalist way of production. However, in so far as the duality itself exists as a matter of fact over a wide range of economic activities, it is still important for economists to take it for granted as such and try to analyse the problems inherent in it.

The present article is going to be concerned with presenting a model of dual economy under a given body of technology of production, and shedding light on the problems of employment and capital accumulation. The model adopted here is highly abstract, so that any statistical reference to reality is left untried, but it would be worth while mentioning that the following model is primarily related with under-developed economies densely populated.

II *Backgrounds of Model and Assumptions*

Let us begin our discussions with dividing an economy into two sectors, namely the capitalist and the subsistence sectors. The capitalist sector is the sector production of which is under the rules of capitalist methods, namely in which all the means of production are owned by capitalists, capitalists endevour after realizing maximum rate of profit on the amount of capital invested, and labourers earn their livings as wage-earners. The subsistence sector is, on the contrary, the sector in which workers earn their livelihoods by their own labour without or, if any, with a negligible help of capital goods (though, sometimes, land is essential factor of production on which labour is worked), and the motive of workers to run production is simply to earn the income to support the members of their families at least at the subsistence level of living. This latter motive should be most clearly contrasted with the motive of capitalists who strive for the maximum profits on the amount of capital invested.

As a matter of fact, presumably, we may include not only agriculture but also small industries such as cottage industries into the concept of the subsistence sector, but by doing this, we may fall into the well-known difficulties of index-number problem. In what follows

we shall visulize this model by supposing the capitalist sector as industry and the subsistence sector as agriculture. In urban districts, manufacturing industry is under the regime of capitalists, and the necessary labour forces are mainly supplied from rural districts. In rural districts, however, agriculture is carried on by small peasants who borrow the farm-land from land-lords and consume their annual income (after paying rents on land) within their families. If any unemployment happens in urban districts, men unemployed may return to farm, because the farm is a mother land for them.

It is unnecessary to impose such a restrictive limitation to our model that there is no circulation of outputs between these two sectors. Workers in agriculture may exchange a part of their outputs for the outputs produced in the capitalist sector, and also labourers in industry as well may buy the outputs produced in the subsistence sector. However, this sort of exchange-problem will complicate the matter considerably, so that it is wise to assume that, for the sake of simplicity, prices of all commodities are constantly given. We shall keep this assumption throughout this paper.

For simplicity, if we could assume that land is unnecessary in the capitalist sector as a factor of production, the whole product is distributed among capitalists and labourers without residual, and also if we could assume that capital to start work is unnecessary in the subsistence sector, whole product is distributed among land-lords and workers without residual. In what follows, we shall maintain the assumption that entrepreneours are identified with capitalists.

III *Mechanism of a Stationary State*

Our first task is to make clear the mechanism of economy in a stationary state in which (1) the number of working population, (2) the volume of capital and (3) the state of technology are all maintained constantly. Perhaps we should further add that the volume of land is also given constantly.

(1) Production in the Subsistence Sector

In general it may be reasonably assumed that the average product of worker in the subsistence sector is, other things being equal, a decreasing function of the number of worker owing to the law of decreasing returns of land. To make the matters as simple as possible, we assume that only one kind of consumption-goods, say corn, is produced in this sector, that the given volume of farm-land is owned by land-lords, and that farmers borrow farm-land by paying rent on it. By its own nature, there is no capitalists in this sector, so that the annual product is distributed wholly among land-lords and farmers.

Let M stand for the number of worker engaged in this sector in terms of one year's labour time and S for the total output of one year in terms of corn. By our supposition of the law of decreasing returns with the volume of land kept constant, we may establish the following functional relationship

$$\frac{S}{M}=E(M)$$

such that $E'<0$, where E' denotes the first derivative of E-function with respect to M (for simplicity we assume a continuous functional relationship). Let us call this the average pro-

duct function in the subsistence sector.

From the average product function as described in Fig. 1, we may easily derive the marginal product function of worker as indicated by a dotted curve in the figure. As is well known, as far as a perfect competition prevails between farmers and land-lords, the annual income of farmer is determined at the marginal product of worker with the difference between the average and the marginal products going to land-lords.

Now we may argue that there exists a certain level of basic standard of living in a society. This is rather a conventional concept which may differ from time to time and from society to society, depending upon historical, social and economical enviroments of the society, and the convention changes as the society develops. In Fig. 2, this is denoted by b.

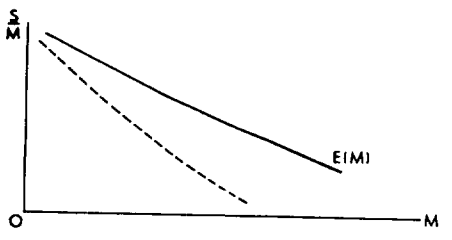

Fig. 1

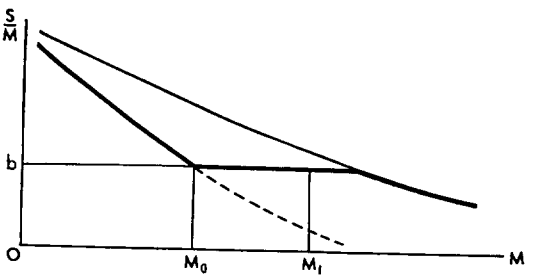

Fig. 2

A problem arises, however, when there appears the marginal worker whose additional output is short of this conventional basic level b. Theoretically even a possibility of negative marginal product of worker might be conceived of. If we accept the Malthusian theory of population, the number of worker corresponding to this basic standard, namely M_0 in Fig. 2, constitutes what we call "Malthusian point" in the sense that if workers increase in number over this point, this causes the number of worker to decrease by means of poverty and if workers decrease in number below this point, the number of workers increase until the basic level of standard is realized.

However, it is hard to whether the number of population is dependent upon the standard of living. It is rather convenient to suppose that the growth rate of population is independent on this standard of living. As for the marginal worker whose additional output is short of this basic level, we may introduce the idea of "residual principle" of rent by supposing either that land-lords do not dare to take off rents so as to make the farmers starve or that law does not permit to do it. In case if the number of worker is short of the Malthusian point, then we may rely upon the traditional marginal principle, namely the annual income of one year's labour is determined at the marginal product. Thus the annual income of farmer is denoted by a stout curve as shown in Fig. 2.

Let us say that there is a *disguised unemployment* of worker in the subsistence sector if there are men whose marginal product is less than the basic level of living standard in the society. In terms of Fig. 2, if the number of workers engaged in this sector is M_1 and the basic output is given at b, the number of disguised unemployment of worker is denoted by M_1-M_0.

There can be hardly any explantation for that one of the major economic problems in underdeveloped economies of the present day is decisively connected with this sort of dis-

guised unemployment. It is sometimes maintained that in some underdeveloped countries even almost one-third of population in agriculture can be removed to other industries with a negligible decrease in the total output in agriculture.[1] If so, we have no hesitation in saying that the marginal product of worker is almost nil in this sector, so that there must be a substantial number of disguised unemployment of worker in agriculture.

Then a question arises. Can we also talk about a disguised unemployment of labour in the capitalist sector? In order to answer this question, we have to make clear the mechanism of determination of wages and profits in the capitalist sector.

(2) Wages and Profits in the Capitalist Sector

For the sake of simplification we shall keep the assumption that, while being removed in the following sections, the capitalist sector forms so to speak a 'closed model' in the sense that the only way for labouring class to make livings is to be wage-earners, so that labourers are obliged to accept any level of wages until they are fully employed. We also assume the perfect competition between capitalists throughout this paper.

In what follows, we shall adopt the simplest case of "point-input-point-output" about the capitalist sector and impose the following assumptions:

(1) this capitalist sector produces only one kind of consumption goods the price of which is kept at unitary level;

(2) the number of labourer is measured in terms of one year's labour and the payment of wages is done in terms of consumption goods;

(3) only labour is the original factor of production;

(4) the economy is in a state of long-run equilibrium in the sense that the rate of profit on the value of capital invested is maximuized and at the same time this rate is equal the rate of interest which is prevailing in the market.

For simplicity let us think as follows. At some period, one year's labour is input for producing trees. Let us suppose that any additional labour is not needed during the process of growth of trees. Trees will grow as time goes by but with a decreasing rate. Let O denote the total number of trees and L the number of labour in terms of one year's labour. Thus $\frac{O}{L}$ is the volume of trees produced per one year's labour in a process of production. Let t denote the period of investment which passes from the moment of labour-input to the moment of tree-output. We may have then the following functional relationship:

$$\frac{O}{L}=F(t) \qquad (1)$$

such that $F'>O$ and $F''<O$. Let us call this the production function. The problem for capitalists is to select the most profitable method of production, namely the period of investment which realises the maximum rate of profit on the amount of capital invested.

It is obvious from the outset that in a state of equilibrium the future value of inputs accounted on the base of the prevailing rate of interest has to be equal the value of output which are obtainable in a future date. Let w be the wage rate in terms of consumption goods per one year's labour and i the rate of interest per period. Under the case of "point-input—point-output" the future value of inputs after t-th period is

$$w(1+i)^t=we^{rt},$$

[1] Second Five Year Plan, Government of India, Planning Commission, 1956.

where r is the instantaneous rate of interest on the base of the natural logarism e. Thus in a state of equlibrium we have

(2) ..... $\frac{O}{L}=we^{rt}$.

The condition of a long-run equilibrium requires further that the rate of profit must be maximised and that the rate of profit thus maximised must be equal to the rate of interest. This is given by the following condition:

(3) ..... $F'(t)=rF(t)$,

where $F'(t)$ denotes the first delivative of the production function with respect to the period of investment.

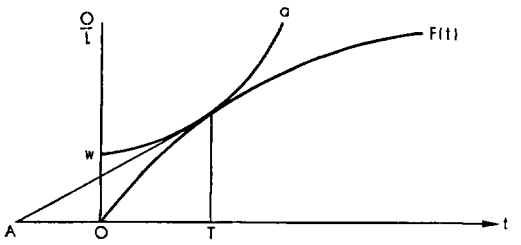

Fig. 3

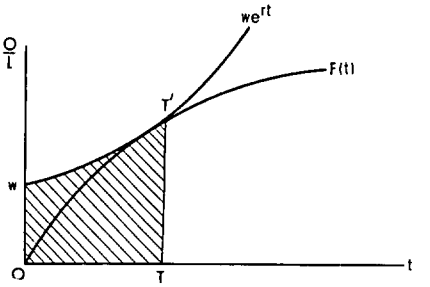

Fig. 4

The above equations (1), (2) and (3) are most clearly shown by Fig. 3. By our assumption the production function (1) is denoted by the curve $F(t)$. Let the wage rate be given at w. The future value of inputs, we^{rt} is denoted by the curve a. At T of the period of investment the rate of profit (therefore the rate of interest) is maximized and this is given by $\frac{1}{AT}=r$.

Let us suppose that the process of production has been synchronized already and that there is a continuous flow of outputs at every moment of time. In this situation, a process of production must be keeping the area shadowed in Fig. 4 as the value of capital per one year's labour, because at every moment one year's labour is input in a process of production and at the same time the volume of output at the level of TT' is being produced.

Let us denote this area by C, then we have

$$C=\int_o^T we^{rt}dt.$$

Let us suppose that at every moment L-processes of production are carried on by L-year's labour forces. Let K denote the total value of social capital. Thus we have

(4) ..... $\frac{K}{L}=\int_o^T we^{rt}dt=\frac{W}{r}(e^{rt}-1)$.

Substituting the equation (2) into this equation, it may be obtained

(4)' ..... $O=wL+rK$.

This last equation is nothing but what we call the distribution side of national product in the sense that the total product O is distributed between labourers, wL, and capitalists, rK. From (4) we may reach to (4)' and from (4)' we may have (4), so that (4) is equivalent to

(4)'.

Let us suppose for a moment that the total value of social capital K is given at $\bar{K}$ and L at $\bar{L}$. Thus it follows

(5) ..... $K=\bar{K}$

(6) ..... $L=\bar{L}$.

Apparently there are six equations for six variables, so that we may determine the variables uniquely.

In Fig. 5, the vertical axis measures both the labour productivity $\frac{O}{L}$ and the wage rate w, and the right and left parts of horizontal axis shows respectively the period of investment and the number of workers to be employed, L. By the equation (1) draw the production function F on the right plane of the figure. Let the wage rate be given at the point α, then the period of production A will be adopted, because at this point the rate of profit is maximised. Since the value of capital in terms of consumption goods is determined by the equation (4), we can immediately determine the number of men employed in terms of one year's labour. Let us dictate it by the point a along the left part of the horizontal axis. Let us coppy a point a' on the left plane which connects α and a. Next let the wage rate be given at the point β. In the same way as before, capitalists will adopt the period of production B, and at the same time the number of workers to be employed is denoted by the point b with the volume of capital kept at $K=\bar{K}$. Again dot a point b' on the left plane in the similar way. If we repeat the same procedure under any given level of wages, we are able to obtain a curve which is indicated by a dotted curve in the left plane of the figure. This is nothing else but the demand function for labour with the volume of capital kept at $K=\bar{K}$.

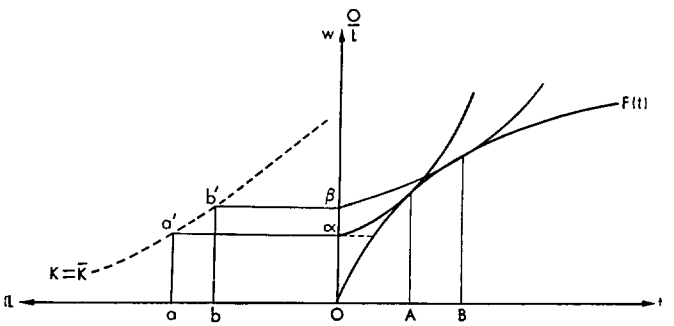

Fig. 5

It will be easily seen that the position of demand function for labour shifts leftward as capital accumulation goes on, because under a given level of wages, the larger the volume of capital the more the number of labourers to be employed. Therefore, we may expect that as capital accumulates the wage rate becomes higher if the number of men to be employed remains constant.

(3) Total Equilibrium in the Economy as a Whole

So far we have been concerned with the analysis of each sector in its 'closed' form. There does not remain any difficulty in combining them into one model and asking the relationship between these two sectors in a state of total equilibrium in the economy as a whole.

The only point to be made here is the relationship of the wage rate in the capitalist sector to the income of working population in the subsistence sector. In general we may

say that the former is larger than the latter, and this discrepancy is without doubt a commonly observable phenomenon in many underdeveloped economies of the present day. It would be explained partly by imperfection of markets due to institutional or geographical reasons, partly by the emotional or phychological hesitancy of workers to work in the capitalist sector and so on. Whatever the reasons may be, we may generally take it for granted that the wage rate in the capitalist sector is higher than the average income in the subsistence sector, and that, although within a certain limit, workers in the subsistence sector forms so to speak a 'non-competing group' against labourers in the capitalist sector. Let s denote the income of a worker in terms of one year's labour. Thus we may have

$$w=u(s)$$

such that $w \geq s$.

To begin with, let us suppose for a moment that s is given, so that we can determine w from $u(s)$. With the level of real wages given, capitalists will adopt the most advantageous method of production among a given body of technology. This leads to the following equations:

(1) ..... $\frac{O}{L}=F(t)$

(2) ..... $we^{rt}=F(t)$

(3) ..... $F'(t)=rF(t)$

(4) ..... $\frac{K}{L}=\int_o^T we^{rt}dt=\frac{w}{r}(e^{rt}-1)$

(5) ..... $K=\bar{K}$

(6) ..... $w=u(s)$.

Since s is given, these six equations are sufficient to determine six variables O, L, t, K, r and w. If s is higher, w will be also higher, so that, other things remaining constant, the rate of profit r will be lower.

In order to determine s within the system, however, we must assume that the total number of working population in terms of one year's labour is given at $\bar{N}$. Then we have the following system:

(7) ..... $s=\bar{E}(M)$

(8) ..... $N=L+M$

(9) ..... $N=\bar{N}$.

In this system, the equation (7) is dictated by a stout curve in Fig. 2. L being already determined, both s and M are easily determined. If the value of s thus determined is different from the prescribed value in the capitalist sector, there will be a movement of labour between these two sectors until the equation (7) is established. In a state of total equilibrium, the above nine equations must be established at the same time, and O, L, t, K, r, w, s, M and N are

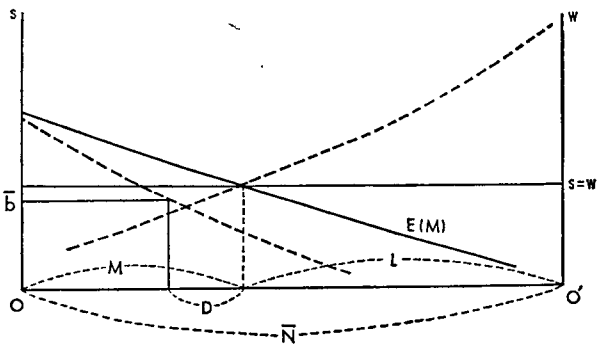

Fig. 6

uniquely determined by these nine equations.

Let us demonstrate the above system by Fig. 6. By the equation (9), the total number of workers in terms of one year's labour is given at N. First we describe the equation (7) so as to make the point o its origin in co-ordinate. In the same way, let us draw the demand function for labour in the capitalist sector with the volume of capital kept at $\bar{K}$ so as to make the point o' its origin. For simplicity let us suppose $w=s$. Thus we can immediately determine the number of workers to be employed in each sector, namely M and L. In this figure working population is living at the subsistence level, we can also find out the number of disguised unemployment of worker, D, in the subsistence sector.

From this figure it must be clear that, within our model, there cann't be any disguised unemployment of labour in the capitalist sector, because the wage rate in the capitalist sector must be at lowest at the level equal to the income of workers in the subsistence sector. If the wage rate were lower than this level, labourers would move from the capitalist sector to the subsistence sector.

So far we have maintained the assumption of constant volume of capital and constant number of working population. Our next task is to develop a dynamic model in the course of capital accumulation and growth of population.

IV *Capital Accumulation and Population Growth*

(1) Process of Capital Accumulation

To make the problem as simple as possible, we shall assume that workers consume the whole of their income on the one hand, and capitalists save and invest their whole profits on the other. As for the income of land-owners, it would be best to suppose that it is wholly consumed considering that we are assuming that capitalists do not consume, namely saving of land-owners, if any, is balanced by the consumption of capitalists.

For a moment, let us assume that the number of working population does not change. Because of that the total profits of capitalists are equal to the rate of profit, r, times the total value of social capital, K, the volume of capital will be increased by the amount of $\triangle K=rK$ every year. Because the amount of capital has increased, the number of men to be employed will be increased if the wage rate does not change. This means that the marginal product curve of labour or demand curve for labour moves from k_0 to k_1 in Fig. 7.

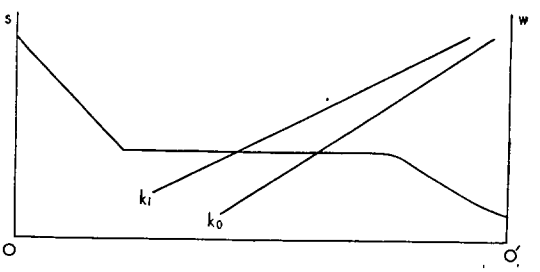

Fig. 7

As long as the state of technology remains constant in both sectors, a shift of the demand function for labour owing to the accumulation of capital will cause the real wage in the capitalist sector to be higher in the long run. As already made clear, a higher wage rate will cause the rate of profit to be lower, and at the same time the working population will move from the subsistence sector to the capitalist sector. And this process will be re-

peated until there is no motive for capitalists to accumulate capital any further.

It may be said that the income of working class in the subsistence sector, namely the equation (7) forms as it were a kind of supply function of labour to the capitalist sector. Indeed, in many underdeveloped economies of the present day, backward industries such as agriculture, village industries, "from-to-mouth" type of merchants make a vast assemblage of the reserved army of labour to the capitalist sector. It is not right to say that this is a mere cyclical phenomenon due to business cycles. In underdeveloped economies densely populated, this is more fundamentally a matter of secular phenomena originated in the whole mechanism of the economy. As long as there exists a big gap in income of worker between urban district and rural district, and so long as capitalists are in the position to control this gap so as to depress the level of wages in the capitalist sector, the wages would not be expected to rise for a long time, and even though they rise, it is only with a very slow speed, in spite of high rate of capital accumulation.

(2) Growth of Population

If it is allowed also for the number of population to grow, we may have the following equation system:

$$\frac{O}{L}=F(t) \quad (1)$$

$$we^{rt}=F(t) \quad (2)$$

$$F'(t)=rF(t) \quad (3)$$

$$\frac{K}{L}=\int_0^T we^{rt}dt \quad (4)$$

$$\triangle K=rK \quad (5)$$

$$w=u(s) \quad (6)$$

$$s=E(M) \quad (7)$$

$$N=L+M \quad (8)$$

$$\triangle N=qN, \quad (9)$$

where q denotes the growth rate of population. Because this system is a difference equation of the first order, we can determine these variables uniquely if the initial values of K and N and the growth rate of population, q, are respectively given.

Then what are the characteristics of this system? Particularly it is important to know the effects of the relation of the capital accumulation to the growth rate of population upon the division of labour between two sectors and upon the level of real wages.

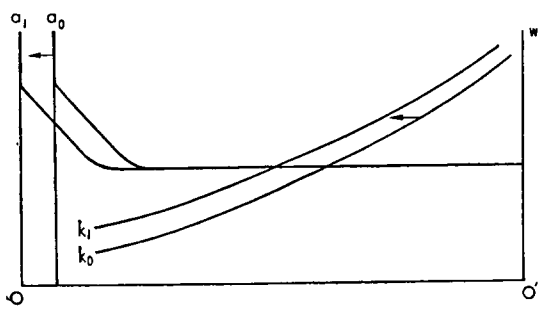

Fig. 8

In terms of Fig. 8, the demand curve for labour in the capitalist sector moves from k_0 to k_1 owing to the accumulation of capital on the one hand, and the supply curve of labour in the subsistence sector moves from a_0 to a_1 on the other. If the shift of demand curve (i.e. k-curves) is relatively larger than the shift of supply curve (i.e. a-curves), the wage rate will be higher, and vice versa. Our question is then to ask the condition which makes

the wage rate unchanged.

In order for the level of wages to remain constant, the following alternative condition must be true. First, in the subsistence sector, the number of worker must be neither on the increase nor on the decrease, namely

$$\triangle M=0,$$

because if $\triangle M$ were positive (or negative), this means that the income of working population in the subsistence sector becomes lower (or higher) after all, so that the level of wages must be on the decrease (or increase). Therefore if there is an increase in the number of working population, this must be all absorbed in the capitalist sector, namely $\triangle N=\triangle L$.

In the capitalist sector, it would be tentatively apparent that for the real wages to be constant the following condition must be satisfied:

$$\frac{\triangle K}{K}=\frac{\triangle L}{L}.$$

Let us denote the growth rate of capital by G_k and the growth rate of population G_n, then it follows

$$G_n=G_k\frac{L}{N}.$$

In this latter equation, $\frac{L}{N}$ denotes the ratio of workers employed in the capitalist sector to the total number of working population. For example, let this ratio be 0.3 and the rate of capital accumulation be 0.1 per period. Then this equation tells us that for the level of real wages to be kept unchanged the growth rate of population must be 0.03. If the growth rate of population is larger than this critical value, the real wages must be on the decrease, and contrariwise if it is smaller than this value, the real wages must be on the increase. It would be almost self-evident for the movement of workers between two sectors to be analysed.

V *Final Remarks*

From such a highly abstract model as this, it is not wise to talk about the realistic figures of underdeveloped economies, still less the issues of economic policy. Our concerns have been only with the analysis of dual economy under the assumption of a given body of technology. In reality, however, there has been a ceaseless stream of 'innovations' not only in the capitalist sector but also even in the subsistence sector. Furthermore, it would be worth while noticing that even in countries poorly equiped relatively to population the problem of effective demand is a crucial matter for the problem of employment, if their effective demand depends largely upon the export of commodities. Indeed export of commodities is one of the most unstable items in effective demand, and capital equipment is sometimes under-utilized by cyclical fluctuations in economic activity depending upon external factors. Perhaps we should discuss the problems of disguised unemployment of labour not only from the long-run point of view but also from the standpoint of short run issues. But these are the problems which are beyond the scope of the present paper.